
Daniele Sepe, Universidade Federal Fluminense

Integrable billiards and symplectic embeddings

The problem of (finding non-trivial obstructions to) embedding a symplectic manifold into another is one of the oldest in symplectic topology and started with the seminal non-squeezing theorem due to Gromov. In dimension 4, many techniques have been developed to shed light on this hard question. Recently, ECH capacities have proved effective in studying symplectic embeddings between subsets of $(\mathbb{R}^4, \omega_{\text{can}})$ called toric domains, i.e. saturated with respect to the moment map of the standard Hamiltonian \mathbb{T}^2 -action on $(\mathbb{R}^4, \omega_{\text{can}})$. Motivated by work of Ramos, which uses complete integrability of the billiard on the disc to obtain some interesting embedding results for the Lagrangian bidisc by showing that the latter is symplectomorphic to a toric domain, this talk outlines how to obtain sharp obstructions to finding symplectic embeddings for some other subsets of $(\mathbb{R}^4, \omega_{\text{can}})$ by relating them to suitable toric domains. These subsets are related to integrable billiards on squares and rectangles. This is ongoing joint work with Vinicius G. B. Ramos.