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On Geometric Quantization of (some) Poisson Manifolds

Abstract: Geometric Quantization is a program of assigning to Classical mechanical systems (Symplectic manifolds and the associated Poisson algebras of C^∞ functions) their quantizations — algebras of operators on Hilbert spaces. Geometric Quantization has had many applications in Mathematics and Physics. Nevertheless the main proposition at the heart of the theory, invariance of polarization, though verified in many examples, is still not proved in any generality. This causes numerous conceptual difficulties: For example, it makes it very difficult to understand the functoriality of theory.

Nevertheless, during the past 20 years, powerful topological and geometric techniques have clarified at least some of the features of the program.

In 1995 Kontsevich showed that formal deformation quantization can be extended to Poisson manifolds. This naturally raises the question as to what one can say about Geometric Quantization in this context. In recent work with Victor Guillemin and Eva Miranda, we explored this question in the context of Poisson manifolds which are "not too far" from being symplectic—the so called b-symplectic or b-Poisson manifolds—in the presence of an Abelian symmetry group.

In this talk we review Geometric Quantization in various contexts, and discuss these developments, which end with a surprise.